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# **Generalization in mathematics at all educational levels**

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## INTRODUCTION

From all processes involved in mathematics, generalization is considered one of the most important ones. For some researchers, generalization *is* what mathematics is about. Thus, whether it is viewed as part of a higher level process, like abstraction or as the core process involved in a particular mathematics field, like algebra, there seems to be an agreement on its significant role in advanced mathematical thinking. This is also acknowledged by most significant curriculum documents, which make an explicit reference on processes related to generalization.

The need for focusing on generalization might be also justified by the development of mathematics as a scientific discipline; this means that arithmetic and computational skills are not enough for the students to ‘grasp’ the deeper underlying structure of mathematics. The teachers should be well informed on that and should be prepared to create opportunities for their students to detect patterns, identify similarities and link analogous facts. But generalization does not appear just by performing the previous activities; to use John Mason’s terms, a shift of attention should take place or, in other words, a shift in the way one sees things.

Contrary to what most people might think, generalization can be even observed in young children; such observations are signified by terms such as ‘early algebra’, which have recently appeared in the relevant literature.

This volume presents various approaches on how generalization is or should be treated in the mathematics classroom. The five parts offer only one way of differentiating between the views presented. Among them the reader may find chapters focused on the theoretical foundations of generalization, but also chapters focused mostly on the implementation of approaches based on generalization, e.g. by pattern recognition. There is a part dedicated to early generalization, in line with the current trends in research that we have mentioned, and another part focused on teachers’ skills in generalizing.

According to John Mason *generalization is the life-blood, the heart of mathematics*; being aware of that fact and being able to accordingly adapt the classroom practices is a highly important aim of mathematics education. We hope that the present volume can offer to mathematics educators and researchers a means to a deeper understanding of the many possibilities existing within the approaches that highlight the role of generalization at all educational levels.

Rzeszow, June 2012

The Editors

